Chassis Torsional Rigidity Analysis
for a Formula SAE Racecar

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Abstract
While the general effects of chassis torsional rigidity on vehicle handling and performance are known, exact guidelines to determine the appropriate chassis stiffness for a given vehicle is not known. This project investigates the effects of chassis torsional rigidity on vehicle handling and performance for The University of Michigan’s Formula SAE racecar with the goal of determining a chassis stiffness design target. Investigations include open loop simulation of steady state cornering and speed trace simulation of transient cornering events using VI-Grade CarRealTime. Simulation outputs including tire forces, vehicle path, and driver inputs are used to determine the limits of acceptable stiffness values. In conclusion a theoretical design target for chassis torsional rigidity is determined that is designable, while taking into account that the design must be buildable and FSAE rules compliant to be used in future Formula SAE racecars. The method with which the value is determined is presented in a way that is reproducible, should the vehicle characteristic change in the future and warrant re-investigation of these effects by future team members.

Introduction
Formula SAE is an international collegiate student design competition focused around the design, manufacturing, testing, and racing of an open-wheel racecar. MRacing, the Formula SAE team at The University of Michigan, Ann Arbor, designs and builds a new car every year. With the design of each new car comes the design of a new chassis. While much of chassis geometry and material choice is dictated by the competition rules, it is possible to build vehicles with a very large range of chassis stiffnesses. Part of each chassis design cycle is the determining of a chassis stiffness target for that vehicle. The purpose of this report is to summarize analysis done during the 2016 design cycle to determine an optimal chassis torsional stiffness design target for future MRacing Formula SAE vehicles.

Literature Review
Studies have been carried out to investigate the effects that the torsional stiffness (resistance to distortion about the longitudinal axis) of a vehicle chassis has on the handling and performance of the vehicle. Although physical testing has shown that vehicles with higher stiffnesses perform better, the exact effects that torsional stiffness has on performance is not entirely known [3]. CAE simulations on the other hand have shown torsional stiffness to not affect the lateral acceleration, yaw rate, or other critical characteristics, during steady state turns [2][3]. The biggest effect of torsional deflection has been found to be the effect it has on lateral load transfer distribution between the front and rear axle [3]. During a steady state turn an infinitely rigid chassis will cause the front and rear roll angle to be identical, as is assumed when suspension
Design calculations are performed. Allowing for twist in the chassis will redistribute some amount of weight transfer between the front and rear tires, causing these values to deviate from their designed values [1]. Because of these effects, the vehicle will handle more predictably if the chassis is stiff enough, relative to the suspension roll stiffness, that the twist can be ignored [1]. To achieve this goal, most racecars design to be 3-5 times as stiff as suspension roll stiffness, while passenger cars design to be 7-40 times as stiff [3]. It can be seen by this extreme range of values, that no consensus has been made as to the appropriate value for chassis torsional stiffness. Finding a standard value against which a chassis can be compared would be helpful in the design of vehicles, but as of current, no standard criterion exists [2].

As well as the effects above, twist in the chassis has also been theorized to have negative effects on the fatigue of the vehicle, suspension characteristic changes caused by relative hard point displacement, and transient response time of the vehicle as a whole to driver inputs [1][2][3]. All of these effects can be easily mitigated by increasing the stiffness of the vehicle, but this has the direct response of increasing vehicle weight. For this reason it is desired to find the lowest value of torsional stiffness that will not decrease the performance of the car.

**Previous Work**

Over the past 30 years, MRacing has taken different approaches to determining the ideal chassis torsional rigidity value for the vehicle. The team has settled on a general rule of thumb of designing the frame, the easiest part of the chassis to modify the stiffness of, to be 10 times the total suspension roll stiffness. It has been assumed that this approach has worked, as past cars designed by this rule have performed exceedingly well at competition compared to other FSAE vehicles during the dynamic events. Along with good vehicle performance, the current generation of MRacing drivers has never indicated that the chassis feels “loose” or is reacting unpredictably to driver inputs, one of the key signs that a chassis is not stiff enough. Using this design method, the 2015 vehicle was designed to have a chassis stiffness of 2100 Nm/deg.

Previously, closed loop full vehicle simulations have been used to try and identify an ideal chassis stiffness target. These simulations have proven unsuccessful, in response the team requested a new simulation methodology to answer this question. The origin of the motivation for this investigation is in the desire to find a simulation method that is successful in providing usable results.

In past design cycles physical testing has been done in an attempt to determine the total chassis torsional stiffness. This testing, as shown in Figure 1, involved constraining the rear spindles, and applying a moment about the front spindles while measuring deflections. During these tests the springs were replaced with solid links to remove the effect of suspension roll stiffness on the measurement of chassis stiffness.

![Figure 1: Chassis stiffness physical test setup](image-url)
Methods
In order to determine the effects of chassis torsional stiffness on vehicle performance and handling, full vehicle simulations were used to simulate steady state and transient cornering events. The tire forces and driver inputs were used as outputs from the model to measure the effects of various chassis stiffnesses. A comparison was done as these output values deviated from the values for an ideal car with no chassis twist. Finally, these deviations were measured and a safety factor for manufacturing error was applied to determine a final ideal design target for chassis torsional rigidity for the MRacing Formula SAE vehicle.

The method for finding an ideal stiffness focused around finding the least stiff the chassis could be before major performance or handling losses would arise. This method was chosen because stiffness is so closely related to vehicle weight, and for a Formula SAE vehicle, weight is the most sensitive parameter on the car. Thus the lower the stiffness can be made, the lower the vehicle weight will be, which is known to significantly increase vehicle performance.

Simulation Model
All of the full vehicle dynamic simulations run for this analysis were done using VI-Grade’s VI-CarRealTime (VICRT). VICRT is a full vehicle real time simulation software that takes in various input parameters about a car and can simulate the performance of the car in both open loop tests, and closed loop tests with a computer driver. The model of our vehicle in this software has been developed by the MRacing team over the past few years. The model includes weights and inertias, suspension geometries, tire models, aerodynamic models, engine power and torque models, and transmission models. The software includes the ability to include chassis torsional stiffness as a variable, or it can run the simulations with an infinite stiffness assumption.

Steady State Turns
Torsional stiffness comes into play in vehicle dynamics as the vehicle experiences cornering events. For this reason the first step in understanding the effects of chassis stiffness was to simulate vehicles of varying stiffnesses in steady state cornering events. As the vehicle goes around a turn weight transfer occurs from the inside tires to the outside tires. For a corner of constant radius being taken at constant speed, the amount of weight transfer is given by Equation 1, where $\Delta W$ is the weight transfer due to corner, $W$ is the weight of the vehicle, $A_y$ is the lateral acceleration of the vehicle, $h$ is the height of the center of gravity, and $t$ is the track width [1].

$$\Delta W = \frac{W \times A_y \times h}{t} \quad \text{(Eq. 1)}$$

As can be determined by this equation, the total amount of weight transfer for a given vehicle of a set geometry cannot be changed. For our vehicle, this weight transfer is not evenly distributed between the front and rear tires. This uneven distribution of weight transfer is referred to as the total lateral load transfer distribution.
The total lateral load transfer distribution, or the amount of weight transfer handled by each tire, can be changed by changing or “tuning” the relative roll stiffnesses of the front and rear of the vehicle. As the weight transfer on the rear or front tires increases, the magnitude of the normal force measured at that tire will increase as well. If this increase in normal force occurs on the front tires, the tires will have a decrease in mechanical grip, leading to an increase in the understeer of the vehicle. This means that for a given steering angle and speed, the vehicle with more weight transfer handled by the front wheels will take a path with a larger radius.

This difference in weight transfer between the front and rear tires creates a moment along the longitudinal axis of the car. It is assumed when determining vehicle dynamics characteristics that the chassis does not react to this moment. In reality, even the stiffest chassis will react slightly to this moment, wanting to twist about its longitudinal axis. By simulating steady-state cornering events and allowing for twist in the chassis the effect of this twist can be determined.

For these simulations corners of three different steer angles and three different speeds were chosen to replicate corners likely to be seen on track. The vehicle was programmed to start at a speed of 15, 20, and 25 m/s. The vehicle was then told to ramp the steer angle from 0 to 30, 20, and 10 degrees respectively, over the course of 1 second. The vehicle then held this steer angle for the remaining 10 seconds of the simulation. These three turns, referred to as tight, mid, and wide, respectively, were repeated for vehicles of various chassis stiffnesses. The normal forces from all four tires was then extracted from the simulation.

After analyzing these initial results, it was apparent more simulations were required to analyze the effect of changing roll rates. For these simulations the same three tracks were used as defined by the same three vehicle speeds and steer angles as before. In this set of simulations, a baseline was run with the car having the baseline 2100 Nm/deg torsional stiffness. Then the vehicle was modified to have a 210 Nm/deg stiffness to represent the extreme of a loose chassis. The effect that the roll rates of the front and rear suspension have on the vehicle was then investigated by decreasing the front, and increasing the rear, spring rates by the same amount over a range of magnitudes. As was done previously, the tire forces were extracted from these simulations to be analyzed.

**Transient Turns**

As the car never actually reaches a steady state on track, the next step in the investigation was to analyze how the vehicle performs during the transient period when entering and exiting a turn. To view the behavior of the car in a transient situation, a simulation method referred to as “speed trace” was used to incorporate aspects of both closed loop and open loop testing.

To create the speed trace, first a baseline car is simulated around a J-turn. For this model, the baseline car was the standard MRacing vehicle setup, but with chassis twist ignored by the simulation software. The model then implements a closed-loop computer driver to take the baseline vehicle around the specified turn as fast as the computer determines is possible. For
these simulations, once again a tight, mid, and wide, set of J-turns was used. These turns simulate turns likely to be seen on track. Once the vehicle had completed the closed loop run the position and velocity vectors of the baseline lap are extracted. These positions and velocities are used to form the new input for the other models. As the car is modified from its baseline set up, each modified car is instructed to attempt to drive the same path with the same velocities as the baseline model. As the car attempts to drive the same path, the driver inputs required by the simulated car are extracted from the simulation. By using the required driver inputs as the simulation output, vehicle performance is measured by evaluating which car is easier to drive. By using the speed trace method all the vehicles should have nearly identical lap times and accelerations, but they will all require a different set of driver inputs to perform in the same manner. The vehicle that responds best to the simplest set of driver inputs will be inherently better as it will be easier to drive at the vehicle’s limit and will respond more predictably to driver inputs.

Results

Steady State Turns
A plot of the normal forces on the outside tires for the various simulated vehicles is shown in Figure 2. The mid corner was omitted in the graphs for simplicity, as it followed an identical trend as the more extreme tight and wide turns.

![Figure 2: Front and rear outside tire normal forces for tight and wide turn at various stiffnesses where Actual stiffness refers to 2100 Nm/deg.](image)

As can be seen in Figure 2, the tire forces in the front increase and the tire forces in the rear decrease, as the chassis becomes less stiff. This effect can be explained by the way the chassis reacts to the moment created by the distribution of weight transfer. As a moment is created along the longitudinal axis of the car, the chassis begins to twist, this allows for weight transfer to move from the side of the car handling more, to the side of the car handling less. In the case of the MRacing vehicle, this means as the chassis twists more, weight transfer should be moved
from the rear to front, increasing the front tire normal forces as is seen in the simulation. This increased load on the front tires increases the understeer gradient and causes the vehicle to take a wider turn for the same given driver inputs.

The initial steady state simulations show the vehicle to only be affected by chassis stiffness because of the way chassis stiffness changes the distribution of weight transfer between the front and rear. If this is true, the effects on the vehicle due to a loose chassis should be reversible, by accounting for the chassis twist. If the front roll stiffness is decreased, and the rear is increased, the vehicle will have a larger difference in the front and rear total lateral load transfer than before. With the assumption that the chassis can’t twist, this would mean the vehicle would have different tire loads in the same steady state turns, as the baseline vehicle. But if chassis twist is taken into account, the redistribution of weight transfer from the rear to the front, as the chassis twists, will bring the car back closer to its baseline behavior. To test this effect, the vehicle was simulated once again given the same inputs. A baseline vehicle was set with our 2015 design target chassis stiffness of 2100 Nm/deg. The stiffness was then decreased by an order of magnitude and the test was run again. Finally this loose chassis was run with varying spring rates in the front and rear. The plot of the front outside tire normal forces for these simulations can be seen in Figure 3.

![Front Right Tire Forces For Variable Roll Rates on a Wide Turn](image)

Figure 3: Front outside tire normal forces for chassis of various chassis torsional stiffnesses and of various suspension spring rates. Where Delta # refers to a # lb/in spring rate decrease in the front springs and # lb/in increase in the rear springs.
As can be seen in Figure 3, by increasing the difference in roll rates between the front and rear, steady-state vehicle behavior of the actual and loose chassis can be made identical. This means that for steady state events chassis stiffness is inconsequential, as the looseness in the chassis can be tuned-out by modifying front and rear roll rates.

**Transient Turns**

A plot of the steering angle against time for the various simulated vehicles is shown in Figure 4. The tight and mid J-turn was omitted in the graphs for simplicity, as they follow an identical trend as the more extreme wide J-turn.

![Steering Angle with Time for Differing Stiffnesses](image)

Figure 4: Steering angle is shown against time for the wide J-turn event for vehicles of varying chassis torsional stiffnesses.

As can be seen in Figure 4, the magnitude of the steering events increases as the chassis becomes less stiff. Along with magnitude change, the input maneuvers have a phase shift and become more delayed as the chassis becomes less stiff. These same trends hold for throttle control and brake control, but are easiest to see by analyzing steering angle.

These steering angle magnitude increases and phase shifts indicate that the car will become more difficult to drive as the chassis becomes less stiff. If the magnitudes of the first steering angle peak are plotted against chassis stiffness a clear exponential decay trend appears, as can be seen in Figure 5.
Figure 5: Maximum steering angle in the wide J-turn event against chassis torsional stiffness

A trend of this form holds true for the other features of the inputs as well, but can be best seen with this example. As shown in the graph, a theoretical ideal value is approached as the simulation ignores chassis twist all together. As the vehicle deviates from that ideal value it becomes harder to drive. As can be seen by the phase shift as the chassis acts more like a spring, the vehicle has a delayed reaction to the driver inputs, and the driver must increase the magnitude of the inputs to compensate. On track this would directly correlate to a car that is much harder to drive and control. For this reason, some deviation from this ideal value is acceptable, but not too much as to allow the chassis twist to have a major, or driver noticeable, influence on the vehicle behavior. A reasonable cut off point is that where the trend begins to have major deviations from its linearity and begins to exponentially grow as the chassis loosens. This transition is seen around the 10% deviation mark from the ideal value. At this point, increased chassis stiffness enters a zone of diminishing returns, and decreased chassis stiffness begins to enter a zone of non-linear behavior. This puts the allowable chassis stiffness at approximately 1550 Nm/deg.

Discussion
Along with the results from the vehicle simulation, other factors of the design of a Formula SAE vehicle must be taken into account. Competition rules are a major driving factor behind the final
vehicle chassis stiffness. Rules requirements for energy absorption and deflection of the chassis measured through 3 point bend tests, and perimeter shear tests, dictate the range of acceptable structures for monocoque vehicles. This means that for a given geometry, there is a theoretical stiffness floor where the car cannot have a lower stiffness through material removal while still being rules compliant.

Another major influence on chassis stiffness is the quality and manner of manufacturing. Errors or deviations in the manufacturing method can lead to actual chassis stiffness values that differ from design values. Physical spindle-spindle test from previous vehicles have shown that differences in design, analysis, and manufacturing can lead to measured stiffness 10-20% off from the designed value. If this manufacturing error is taken into account and applied to the previously established base value of 1550 Nm/deg as a safety factor, a new minimum chassis stiffness of around 1900 Nm/deg can be found. Designing to a stiffness less than this value puts the vehicle at risk of being significantly affected by chassis twist once manufacturing is complete.

Finally, it is important to note that this entire analysis was done via simulation methods. Chassis stiffness most prominently affects the way the vehicle reacts to driver inputs, and the way the driver is able to feel the response of the vehicle. These effects can only truly be determined by testing a built vehicle on track with the drivers who will be driving it in competition. A good indicator that the method employed by this analysis is appropriate, is that the chassis stiffness value determined in simulation (1900 Nm/deg) is very similar the chassis stiffness design value determined via 30 years of iterative design (2100 Nm/deg).

**Conclusion**

In conclusion, from the simulation of both steady state and transient turns we find that chassis stiffness significantly affects vehicle behavior and handling. As the chassis gets less stiff the vehicle behaves in an increasing undesirable manner to driver inputs, and becomes increasing hard to drive. Through the analysis of these trends, with manufacturing error taken into account, we can conclude that future MRacing vehicles should be designed for the lightest chassis possible that has a torsional stiffness greater than 1900 Nm/deg. Stiffnesses greater than this value result in added weight for diminishing returns, while stiffnesses less than this value put the car in danger of entering a regime in which the stiffness has significant effect on vehicle behavior. This analysis method has shown to be quite effective, and significant confidence is had in the results due to their close proximity to the ideal chassis stiffness developed by the team over 30 years of iterative design. As this analysis is based upon the simulation of a baseline model of our vehicle, it is recommended that this analysis be repeated if the design of future cars leads to a significant change in the parameters of the baseline model. This analysis has been explained in a way in which the repeating of these tests by future team members should be easy and time efficient. The true test of the analysis will be seen as the future MRacing vehicles take to the track against other vehicles of different designs.
References

[1] Race Car Vehicle Dynamics, Milliken, F.
[2] Research of chassis torsional stiffness on vehicle handling performance, Pang, Shu-yi