Forecasting Bifurcation Point and Bifurcation Diagram in Ecological Systems

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Introduction

Bifurcations occur in the dynamics of complex systems, from engineering systems to ecological systems. In this phenomenon, the system stability may have an abrupt and significant change that leads to unwanted stability problems. Therefore, it is important to develop bifurcation forecasting methods to predict critical transitions and prevent catastrophic damages in the system. Applying bifurcation forecasting to eco-systems such as the classical prey/predator and vegetation system is important because transition from one steady dynamics to another can often lead to importantly drastic changes such as extinction of species.

Project Objectives

Ecological systems are subject to drastic changes caused by change of parameters. In this sense, bifurcation points of ecological systems become important because the drastic change of the system’s stability may lead to disasters. The system has the ability to recover from perturbations.

Method Overview

When a large perturbation is applied to a system, the system tends to recover from it. This method takes recovery from perturbations and uses that to calculate recovering rate. When the system is recovering, the recovery rate will slow down as the system gets closer to the bifurcation point, a phenomenon called “critical slowing down” (CSD). Thus by calculating recovering rate at different states, the slowest recovery rate (i.e. where states of the system are closest to a fixed point) can be found, therefore, bifurcation diagram can be predicted.

Algorithm

The basic steps of this method used in the project can be summarized as the following steps:
1. Choose two parameter values, μ₁ and μ₂ (note that different parameters result in different states of a system), for each parameter, calculate states of the system (e.g. amount of rabbits) with respect to time.
2. Assume if a system has two dimensions (e.g. prey/predator model has the amount of rabbits as one dimension and the amount of foxes as another), choose either dimension will result in the same prediction.
3. Therefore, calculate η = dx/dt of the chosen dimension where dx tells how far a point is from bifurcation (the closer, the recovery rate is smaller, CSD), du = μ₂ - μ₁, and dv = dx/dt.
4. Linear fitting η vs. μ and calculate where η = 0 (i.e. point where distance to bifurcation is zero, that is the predicted bifurcation point).
5. Repeat Step 3-4 to obtain predicted bifurcation diagram.

Results and Discussion

The system used in this project to forecast bifurcation is predator-prey model. Predator-prey models are arguably the building blocks of the bio- and ecosystems as biomasses are grown out of their resource masses. Species compete, evolve and disperse simply for the purpose of seeking resources to sustain their struggle for their very existence. [2] Here’s modified model from Hare and Ebanks [3] is used:

\[ \frac{dx}{dt} = ax(1-x) - \frac{a(1-m)xy}{1 + c(1-m)x} - H(x) \]
\[ \frac{dy}{dt} = -dy + \frac{b(1-m)xy}{1 + c(1-m)x} \]

Where H(x) = h, a,a,b,c,d,h,m are positive real parameters. In this model, x represents prey and y represents predator.

Set parameters to a=0.6, b=0.5, c=0.1, d=0.3, m=0.1, h=0.1. Then applied method indicated in Algorithm, Figure 3 is obtained:

![Bifurcation Diagram](image)

The results shows a high accuracy in forecasting bifurcation. Note that, data should be measured in pre-bifurcation regime and so predict bifurcation and post-bifurcation situation. One of the problem appeared in the application is when initial conditions for the system are relatively close to actual solutions, the accuracy of the prediction results is increased.

Conclusions

Forecast bifurcation of a predator–prey system has significant importance on prevent extinction or overpopulation of a certain species and further more benefit the whole ecological system. The method provided here has high accuracy in forecasting bifurcation of this model. Even if only several data points are picked in the pre-bifurcation regime, the method gave an accurate prediction. This advantage allows people to only collect easy-to-measure data therefore increase the feasibility of the forecasting.

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References